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Cooperative Learning on Mathematical Problem Solving: Reflections by a Traditional Teacher and Her Students

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COOPERATIVE LEARNING IN MATHEMATICAL PROBLEM SOLVING: REFLECTIONS BY A TRADITIONAL TEACHER AND HER STUDENTS

A Thesis Presented

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ABSTRACT

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As our society becomes more technologically complex, the educational system preparing our students to become citizens of this society must adapt to meet changing demands. Mathematical literacy of the 21st century will require a different model of mathematics education than that which served in the past.

This thesis argues for a model of mathematics education which includes as key components: problem solving, question posing, cooperative learning, concrete manipulatives, and teaching for thinking. This new model sets forth guidelines for a facilitative approach to the teaching of mathematics as opposed to the more traditional, authoritative model. This facilitative model is based on the constructivist view of learning and is presented in contrast to one based on the behaviorist view.

Ultimately, it is in the mathematics classroom that any changes in the mathematics educational system must be played out. The author discusses her implementation of a series of lessons with seventh graders, which incorporated the key components of the facilitative model listed above. The focus is on the changes required of the teacher and the difficulties encountered by a traditional teacher attempting to move toward a more facilitative classroom.

Issues of sharing classroom control, of student - teacher interaction, of curriculum design, of lesson planning, and of functioning within the framework of a traditional school are delineated and reflected upon. Recognition of the intrinsic difficulty of the change required is key to the development of a view of the teacher as a reflective learner. The teacher in the classroom must recognize the ongoing process of growth and change required to remain an effective facilitator of learning and must continually search for unique ways to support that growth in both self and others.

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The Innumerate Citizen

As our society has become more technologically complex, the need to be able to understand and deal effectively with problems that arise in and out of the workplace has called for the attainment of more complex mathematical skills. Most students leave our educational system lacking the necessary mathematical skills to function effectively in this new world. Too many students graduate from high school with too little mathematical training together with a firm belief that the attainment of more mathematical skill is personally unnecessary or impossible.

John Allan Paulos defines "innumeracy" as "an inability to deal comfortably with the fundamental notions of number and chance" (1988, p. 3). In effect, it is equivalent to mathematical illiteracy. Paulos believes that a large segment of our citizenry is innumerate (p. 3). There are many signs which support this contention.

The results of the fourth National Assessment of Educational Progress given in 1986 revealed that most students were not able to apply learned procedures in problem solving situations. "Students are lacking the conceptual knowledge that would enable them successfully to do the appplications, problem solving, and reasoning items on the assessment" (Brown et al, 1988, p. 246).

Industry, universities and the armed forces are burdened with extensive and costly remedial programs to provide students and workers with the skills necessary for effective participation in their programs. Americans are often in the minority in many mathematically based graduate programs in U.S. colleges. In both mathematics and engineering fewer than half of the doctorates awarded go to American graduates. International studies of accomplishment in mathematics and science indicate that American students do very poorly, especially when compared with those of other countries (National Research Council, 1989).

Paulos contends that the gap between the popular perception of various risks in our society and scientists' assessment of those risks is a clear sign of widespread lack of understanding of the mathematics of chance. These risks include flying in a plane, being bombed by terrorists, and being a victim of certain fatal medical conditions. Additional indicators are the public's susceptibility to stock-market scams, pseudoscientific claims, and reliance on astrological predictions (1988).

There are, obviously, many causes which have led us to our current state. The following sections will look at some of them from the viewpoint of their impingement on the traditional mathematics classroom.

The Traditional Model of Mathematics Teaching

The current organization of our schools was created during the industrial age. Students were provided with the skills necessary to work in shops, fields and factories. The average citizen required only a minimum competency in reading, writing and mathematical skills. Advanced mathematics courses were designed for only those few students who would go on to become leaders in business, academia and government (National Council of Teachers of Mathematics, 1989). In elementary school, students learn the basic mathematical skills necessary for everyday life, which in the industrial age was adequate for the problems encountered by most people.

Although "a technology-dominated society requires that everyone have a good grasp of chance, of reasoning, of form, and of pattern" (National Research Council, 1989, p. 46), a mathematics very different from that dealing with the arithmetic algorithms, our educational system has not kept pace with these changing needs. What follows is a look at some of the factors contributing to this lack of response to current needs.

Beliefs About Mathematics. The first factor to be considered is the set of beliefs and attitudes held by our students and the population at large. There is a widely held perception that the mathematics of the past is sufficient: "it was good enough for me, so it is good enough for my child." There seems to be a general lack of recognition that mathematics is not a fixed body of knowledge but rather a changing and growing discipline that can and should be made accessible to most people.

In an article dealing with mathematical problem solving instruction Edward Silver states:

> As a result of their experiences in mathematics classrooms, students develop a set of beliefs about mathematics and mathematical problem solving. ...the majority of junior high and secondary school students believe that mathematics is mostly memorization, that there is usually one right way to solve every mathematics problem, and that mathematics problems should be solved, if at all, in a few minutes or less. (1987, p. 57)

Silver goes on to say that these attitudes and beliefs come about because of a "hidden curriculum" which is unintentional on the part of the curriculum designers and teachers. There is a myth in this country that success in mathematics is only possible for mathematically gifted students. In other countries, where this is not a culturally held belief, students do better and learn more mathematics than their American counterparts (National Research Council, 1989).

In a review of the research on solving addition and subtraction problems and more general problems, Carpenter indicates that students enter school with some fairly sophisticated and appropriate problem solving skills, but that after a few years of formal mathematical training they have abandoned these skills in favor of mechanical techniques (1985, 1984). The instruction students receive actually seems to inhibit their development of effective problem solving skills to use in mathematics applications problems (1985, p. 37).

Teacher-Student Interaction. A second factor to be considered is the interaction between teacher and student in the classroom. Most mathematics classrooms are structured for listening to and imitating the authority at the front of the room. Students are taught rules and procedures to follow, but often have no clear conception of the underlying concepts. An example of this from the author's own experience will clarify this issue. Most of her students have been carefully taught how to perform all of the arithmetic algorithms in elementary school, but, when asked to make up stories based on real life situations where multiplication or division would be required, many students cannot create a realistic problem or scenario. The components of what is hereafter called "the traditional model" of mathematics instruction are clearly delineated in the Mathematics Framework for California Public Schools to show the ineffective model of mathematics instruction which requires change. This model is characterized by "Teaching Rules and Procedures" and it:

> Emphasizes recall
> Teaches many rules
> Develops fixed or specific processes or skills
> Identifies sequential steps
> Is used for specific tasks or situations (limited context)
> Is learned more quickly but is quickly forgotten
> Is easy to teach
> Is easy to test
> (California State Department of Education, 1985, p. 13)

The above model identifies the characteristics of all the mathematics classrooms in which the author has sat as a student and consequently has served as the only known model for her teaching. This leads to a look at a third significant factor, teacher training.

Teacher Training. Teachers continue to teach as they have been taught. For most of the teachers in the classroom today, that was "in the authoritarian framework of Moses coming down from Mt. Sinai" (National Research Council, 1989). The pool of people who somehow do get through our system of education enjoying and feeling competent in mathematics seldom become involved in secondary school education never mind elementary education or the training of elementary school teachers. Most elementary school teachers are least comfortable in mathematics of all of the subjects that they are expected to teach. They learned mathematics as a set of rules and procedures and pass on this belief and their discomfort with mathematics to many if not most of their students.

Mathematics Texts. A fourth factor contributing to the present state of mathematics instruction is the intensive reliance on the textbook as the definition of the mathematics curriculum. Most of mathematics instruction involves the study of specific texts and problems that seem to have no connection to the real world. These texts seldom if ever create situations that are akin to the way real people use mathematics to solve problems. The experiences of the mathematician and others who use mathematics as a tool are not demonstrated in the texts. A great percent of the material presented at any grade level is a review of that which has come before (Usiskin, 1990). Little is present in the texts to help students to become mathematical reasoners, communicators and problem solvers. Many teachers do not feel comfortable generating lessons that deviate substantially from those presented in the texts. Again teachers are inclined to teach the way we have learned.

Standardized Testing. The final factor to address is the effect standardized assessment tests have on the perpetuation of the current situation. Most mathematical assessment in this country has been based on multiple choice testing. Since the test format tends to indicate what outcome is expected from instruction in mathematics, the myths about what mathematics is and does are reinforced. These tests tend to stress that the purpose of mathematics is to solve problems with one right answer at a rapid pace. That originality of thinking and expression, or creativity might have a function in mathematics is nowhere indicated to or evaluated in the test taker.

Effect on Author. The preceeding is a brief analysis of what will be referred to herein as "the traditional model" of mathematics instruction and the contribution it has made to the current state of innumeracy in our society.

The author entered teaching ten years ago with no other model to emulate. She became frustrated with the results of some of her experiences in the classroom. Students often expressed dislike of math. They viewed mathematics as a repetitive, boring subject that they were told was important, but which seemed to have no relevance in any meaningful way in their own lives. Even the students (and most particularly female students) who had above average aptitude and success in mathematics, did not see their futures as centering on developing and using their mathematical skills. They seemed unaware of the opportunities for interesting and creative work available to those who had advanced mathematical skills. Less able students seemed to be able to master the "rules" of the current unit well enough and long enough to pass a test but there was little retention of knowledge nor understanding of why this skill was even being taught. Since the author used the textbook as a guide to curriculum, there being no other source available when she started her job with minimal training, she often could not shed much light on the purpose of certain aspects of the curriculum. The units seemed somehow disconnected and fragmented.

Thus began the search for a possibly different and better way. It has become apparent that she is far from being alone in this search. As the recognition of the crisis in mathematics education has become more widespread, many solutions have been suggested for its improvement. From her studies the author sees a new model of mathematics education emerging.

The Effective Citizen of the Information Society

The National Council of Teachers of Mathematics (NCTM) indicates that any effective system of education must be able to provide the following in the 21st century:

1. Mathematically literate workers: These are workers who can understand the complexities and technologies of communication, ask questions, assimilate unfamiliar information, and work cooperatively in teams. These workers must be capable problem solvers. They must have the ability to understand and define problems, the knowledge of and ability to use different problem solving strategies, the ability to see the applicability of mathematical ideas to problem solutions, belief in the utility and value of mathematics, and the ability to work with others on problems.
2. Lifelong learning: The schools need to provide students with the foundations necessary to change and enter retraining programs as the workplace needs change and grow.
3. Opportunity for all: Equity in schooling has become an economic necessity. Since most job opportunities require more advanced skills, society cannot afford to have only a well

educated minority.

4. Informed electorate: Political and social decisions involve increasingly complex technical issues. Citizens must be able to read and interpret complex and sometimes conflicting technical information (1989, pp. 3-5).

A New Model of Mathematics Teaching

The central component of any new model must be that of teaching for understanding of fundamental concepts rather than the memorization of rules and procedures. Again the basic outline for the new model is taken from the California Framework:

> Emphasizes understanding
> Teaches a few generalizations
> Develops conceptual schemas or interrelated concepts
> Identifies global relationships
> Is adaptible to new tasks or situations (broad applications)
> Takes longer to learn but is retained more easily
> Is difficult to teach
> Is difficult to test
> (California State Department of Education, 1985, p. 13)

The curriculum standards specified as essential for future mathematics instruction at all levels are Problem Solving, Communication, Reasoning, and Mathematical Connections (NCTM, 1989). The new classroom should find students doing, talking about, playing with mathematics. A student's knowledge emerges from experiences with problems. "In sum, ...learning should be guided by the search to answer questions--first at an intuitive, empirical level; then by generalizing; and finally by justifying (proving)" (NCTM, 1989, p. 10). The kinds of questions referred to here are not the typical procedural ones with one correct answer that are asked in most mathematics classrooms, but questions that can only be answered by investigation on the student's part, questions that arise for the student in pursuing an investigation, or questions asked by the teacher to probe a student's thinking. Jeremy Kilpatrick would call this kind of question "substantive" (1987, p. 1).

Kilpatrick gives one view of an alternative, investigative approach which he describes as "do-talk-record" (1987, p. 2). In this method, students are presented either individually or in small groups with various activities which lead them to explore some mathematical idea; then, they are asked to discuss what they have done by sharing what they have found or deciding on a plan for further exploration; and, finally, they are asked to write a report on the outcome of their investigation.

> The point of the activity is to provide an opportunity for students to discern mathematical patterns, develop a system for investigating notation for recording those patterns, formulate their findings into a generalization, and then attempt to test the generalization and find a plausible argument for its validity--in other words, to <u>do</u> mathematics. (Kilpatrick, 1989, p. 2)

In the investigative method of instruction the governing belief is that students can better learn mathematical concepts by being given the opportunity to develop and to use their skills in meaningful problem contexts. Part of their learning of mathematics should find students engaged in solving interesting, open ended problems in small teams and following up their successful solutions with activities that ask them to reflect on the problem solving process itself. This gives students the opportunity to communicate their mathematical ideas. In the give and take of the cooperative group, misconceptions are more likely to be aired and challenged by other group members. Since concepts are developed in context, "students develop a framework of support that can be drawn upon in the future, when rules may well have been forgotten but the structure of the situation remains embedded in the memory as a foundation for reconstruction" (NCTM, 1989, p. 11).

This new classroom makes new and different demands on student AND teacher. As the students working in small groups are encouraged to change their role from that of a passive receiver of knowledge to that of a communicator and mathematical reasoner, there is also a change in the teacher's role. The teacher is no longer just the dispenser of knowledge but serves more as a coach or facilitator when a team's progress gets bogged down. Also by listening in on group discussions or asking students questions which reveal their thinking the teacher can gather valuable information about misconceptions and ideas not yet fully developed in the students' minds. The teacher can use this information for planning further experiences and lessons. The students' roles become active and interactive; the teacher's role less directive, more supportive, monitoring, observational.

For the remainder of this thesis this new model of instruction will be referred to as facilitative. The next chapter will explore the theoretical basis for this model as a more effective method to develop students' mathematical thinking.

CHAPTER II

THEORETICAL BASIS FOR THE FACILITATIVE CLASSROOM

Theory of Learning Underlying New Model

This chapter contains a discussion of the theoretical basis for the new mathematics classroom. The difference between the traditional authoritarian classroom and the facilitative classroom identified in Chapter I can be better understood by comparing two theories of learning: behaviorism and constructivism.

Behaviorism. In the following discussion behaviorism is used as a broad inclusive term to describe any theory of learning that views knowledge as an entity existing outside the learner which is internalized due to interactions with the environment (Schoenfeld, 1987, pp. 2-8). In the behaviorist view the mind is a blank slate and learning consists of the taking in of experiences through the senses and filling up the mind. What is outside of the knower becomes internalized by a repetition of experiences. If the teacher wishes to teach, he/she presents information and the child takes it in. This type of theory has led to two basic types of pedagogical approaches to the teaching of mathematics, the "drill and practice" approach connected with the associationist view of learning (Schoenfeld, 1987, pp. 2-3) and the programmed instruction approach (breaking a task to be learned into small carefully sequenced steps for the learner to master) used by "radical behaviorists" such as Skinner (Schoenfeld, 1987, p. 5). In general, the behaviorist approach views learning as making a connection between stimulus and response and relies heavily on reward and punishment to mold behavior. The learner is rewarded for the "right" behavior and "punished" for the wrong behavior. This serves to increase the "associations between sets of stimuli and the responses to them" (Schoenfeld, 1987, p. 2). In this approach learning is viewed as memorization or rote input of skills and procedures. It is the basis underlying most traditional instruction.

> In traditional instruction, children are assumed to learn by internalizing knowledge; therefore teachers simply correct the errors and present right answers, believing that the learner will then absorb this wisdom. (Kamii, 1985, p. 49)

Schoenfeld also describes this model as the <u>absorption model</u> of instruction (1987, p. 25). The underlying assumption is that if the message does not take on the first or second try, the student will eventually "get it" if the teacher repeats it enough times and in slightly different ways.

As any classroom teacher knows, however, this often is not how it works for his/her students. In addition, most teachers are frustrated by the fact that their students also "quickly forget" mathematical knowledge and skills acquired in their classrooms. It is common to find that students don't recall skills supposedly mastered in previous years or even in the same year when use of the skills would be appropriate. An explanation of this phenomenon is suggested by research in cognitive psychology. Bransford declares it to be a function of "inert knowledge" (1986, p. 1080). This is knowledge that a person does have available (stored in long term memory), but does not access in many situations where its use would be of benefit. His discussion indicates that traditional educational practice creates this phenomenon.

Theories of memory retrieval state that if the situations where procedures would be useful are not stored in the brain at the time the procedures are acquired, then the procedures will not be reliably activated when the situations arise that could benefit from their application (Howard, 1983, p. 162). Much of the knowledge acquired in mathematics class is stored in such a way as to be only keyed by mathematics class experiences and sometimes not even there if the circumstances of the new use situation are not sufficiently close to the experiences under which the information was stored. Modern theories dealing with learning as the acquisition of organized knowledge structures focus on "conditionalized knowledge" which means knowledge that includes information about the conditions and constraints of its use (Bransford, 1986). Even after several attempts to teach students how to add fractions over successive years and even if the student can perform the algorithm correctly on one day, many students forget if asked to add fractions several weeks later. This is only one example of many that could be cited as normal classroom experience. Piaget offers us another view of learning that appears more promising to explain events such as these.

Constructivism.

> Jean Piaget proved, however, that children do not acquire knowledge directly by internalizing it from the environment. With more than 60 years of scientific research he proved that children construct knowledge from the inside through interaction with the environment, by going through one level after another of being "wrong" from an adult's point of view. (Kamii, 1982, p. 1)

The constructivist viewpoint says each of us naturally constructs ideas about the world as an interaction between our experiences and our stage of mental development. This development is biologically determined and evolves in each individual as a function of maturation and experiences. Thus children learn not by an accumulation of ideas but by modifying old ideas. Real learning can only take place if the individual is presented with an experience that creates dissonance between ideas already held and new ideas.

Piaget separates knowledge into three categories: physical, social, and logico-mathematical. Physical knowledge is knowledge taken in about physical properties that actually exist in objects in external reality. Social knowledge deals with conventions worked out and agreed upon by people. The red in the feathers of a cardinal is an example of physical knowledge. The word "red" in English to describe that color is an example of social knowledge. Both of these kinds of knowledge can be transmitted or taught in the traditional way as a transmission from the teacher to the learner, although if a person is not developmentally ready or lacks the receptors to perceive the redness, even these kinds of knowledge cannot be transmitted. Piaget's theory contends that the third kind of knowledge cannot be transmitted but must be constructed by each person; that, in fact, no one can "teach" anyone else this kind of knowledge.

Logico-mathematical knowledge consists of relationships constructed by each individual. It is not something that exists "out there" but rather is a result of mental functioning. Its source is internal to the individual. Perceiving "difference" between two objects would be impossible if facts about each object remained isolated, stored pieces of information. The mind is able to construct a relationship from those stored facts. "Different" is an internal, constructed entity that has no discrete existence in the external world (Kamii, 1985).

Piaget viewed mathematical ideas as constructions of the mind. Each individual builds those constructions naturally if developmentally ready and if presented with experiences enabling those constructions. Within this framework of how students learn the teacher's role becomes a very different one.

The Teacher As Facilitator. Within the constructivist view the goal of the teacher is to establish an atmosphere conducive to children's thinking. Although the teacher cannot transmit logico-mathematical knowledge, he/she can influence the student's construction of that knowledge.

> They fuel the child's mental activity by such indirect means as saying something that casts a doubt in his mind about the adequacy of an idea. They also do things that become for him an impetus for making a new relationship. (Kamii, 1985, p. 31)

The teacher's role is not to correct the student when he/she makes an error but to ask the child to explain the method used to arrive at the answer. "He should be encouraged to defend his idea until he decides that another solution is better" (Kamii, 1985, p. 36). Within this framework errors are seen as a natural part of the process of learning. They are expected. As each child is the discoverer of his/her own errors he/she will not suffer the embarassment of not getting the "right" answer. The teacher's role is to create the situations within which the student has the opportunity to encounter alternative points of view which will reveal the incorrectness of his/her ideas.

In the constructivist view students' lack of learning is not just that it has not yet occurred, but will after enough repetitions of experience; but rather that it is more likely that a misconception has been learned which will only be changed if the student is presented with an experience which causes him to perceive his misconception and modify it. The experiences offered by the teacher are, of course, the lessons he/she plans for the students.

Important Components of Lessons and Their Goals

In the following discussion the author has attempted to isolate some of the most important components that would be contained in lessons which fulfill the requirements indicated by the constructivist perspective.

Teaching for Thinking. Certainly one of the most important aspects of any lesson should be that it engage the student in thinking about mathematical ideas. The teacher should give problems or ask questions that engage the student in experiences that help him/her construct mathematical concepts. Arthur Costa "notes that many educators have come to view thinking skills as perhaps the most basic of the basic skills--they are skills that facilitate the acquisition of all other learning" (Costa, 1987, p. 16).

Costa views thinking skill instruction as not an add-on or quick fix, but rather as an integral part of curriculum and instructional practice.

> Teaching for thinking simply means that teachers and administrators examine and strive to create school and classroom conditions that are conducive to children's thinking. This means that:
>
> 1. Teachers pose problems, raise questions, and intervene with paradoxes, dilemmas, and discrepancies that students can try to resolve.
> 2. Teachers and administrators structure the school environment for thinking--value it, make time for it, secure support materials, and evaluate growth in it.
> 3. Teachers and administrators respond to students' ideas in such a way as to maintain a school and classroom climate that creates trust, allows risktaking, and is experimental, creative, and positive. This requires listening to students' and each others' ideas, remaining nonjudgmental and having rich data sources.
> 4. Teachers, administrators, and other adults in the school environmental model the behaviors of thinking that are desired in students. (Costa, 1985a, p. 20)

In addition to teaching FOR thinking, Costa stresses that teaching OF and ABOUT thinking are important if we are to develop in our students the thinking skills they will need in the society of the future. Teaching OF thinking means teaching cognitive skills directly in different subject areas. "Steps in problem solving might be taught directly during math and science instruction" (Costa, 1985a, p. 20). Schoenfeld exemplifies this point of view by helping students to become better problem solvers through the teaching of effective problem solving strategies explicitly (Schoenfeld, 1987).

Teaching ABOUT thinking means teaching students about how the brain functions, about how they learn. It means helping them to step back and think about how they think, giving them opportunities to reflect on thir own thinking and learning processes (Costa, 1985a, p. 21).

Cooperative Learning. Another important lesson component is to include cooperative learning experiences (Davidson, 1990; Dishon, 1988; Johnson et al, 1988; Johns Hopkins, 1986). As noted in the first chapter, one of the skills required of the mathematically literate worker will involve being able to work cooperatively in teams to define and solve problems. Students cannot learn these skills unless opportunities have been provided in school to acquire them. Such a "... perspective suggests that the 'internal dialogues' of competent problem solvers result from their having internalized aspects of the cooperative problem solving sessions in which they have engaged" (Schoenfeld, 1985b, p. 144).

> At a time when being able to interact effectively with other people is so vital in marriages, in families, on jobs, and in committees, schools insist that students don't talk to each other, don't work together, don't

> pay attention to or care about the work of other students--students are encouraged not to care about other students' learning in the classroom. (Johnson et al, 1988, p. 7)

The social interaction which occurs in cooperative groups is also extremely important from the constructivist view of learning. Children must express and defend their ideas to the other members of the group.

> According to the theory of constructivism, children learn by modifying old ideas, not by accumulating new ones. A debate about the superiority of one idea or another is good because it encourages children to think critically by putting different ideas into relationship with one another. It also allows students to modify ideas autonomously when _they_ are convinced that new ideas are better. (Kamii, 1984, p. 414)

Manipulatives. Another lesson component for use in the facilitative classroom involves the use of manipulatives. The NCTM _Standards_ for grades 5-8 state that every middle school "classroom will be equipped with ample sets of manipulative material and supplies (e.g., spinners, cubes, tiles, geoboards, pattern blocks, scales, compasses, scissors, rulers, protractors, graph paper, grid and dot paper)" (1989, p. 87). This reflects an ongoing commitment by the NCTM to the idea that the use of manipulatives is an important component of mathematical learning. All issues of the _Arithmetic Teacher_ carry lessons which utilize manipulatives to enhance the presentation of a concept. Marilyn Suydam (1986) has indicated that the use of manipulative materials in mathematical instruction seems to improve students' mathematical achievement significantly. These findings affect children at all grade levels and levels of ability. Of particular interest here is that use of manipulatives significantly improved problem-solving scores and understanding of equivalent fractions (an important aspect of middle school mathematical content) (Suydam, 1986).

A look at Piaget's work also gives great support for students' learning of difficult abstract ideas being enhanced by exposure to physical contexts. Piaget posits two logical stages of intellectual development: Concrete Operational (logical thinking limited to physical reality) and Formal Operational (abstract and unlimited logical thinking). "Each stage is made possible by those stages preceding it. Earlier understandings are integrated at higher levels of organization and abstraction" (Labinowicz, 1985, p. 15). Although Piaget's developmental stages are linked to age ranges, the thinking level at which an individual of any age functions may be context related.

> We may demonstrate facility with abstract ideas (formal operational thinking) in areas of expertise, while in new areas of experience, like physics, we still need to develop our thinking in physical contexts (concrete operational thinking). (Labinowicz, 1985, p. 16)

Labinowicz thus supports the idea that the learning of complex abstract concepts is strongly supported by first presenting the ideas using physical experiences. Physical models are of great assistance even to adults who function well at the formal operational level when struggling to grasp new abstract concepts.

Student-Teacher Relationship

Teaching/learning, of necessity, involves interaction among the people gathered together in the learning situation. A model for the facilitative classroom would not be complete without a look at those interactions.

Handling Discussion. In creating a classroom conducive to student thinking, the teacher will be concerned about what kinds of things to say. The verbal behaviors used by the teacher in interactions with the students have the greatest impact on whether the classroom is authoritarian or facilitative. Art Costa indicates there "is a relationship between the level of thinking inherent in teachers' verbal behavior and the level of thinking of their students" (1985b, p. 126). The kinds of questions the teacher asks and the way the teacher responds to the students' questions or ideas are the two main ways the teacher has of setting the tone of the classroom. Costa's article was one of the most helpful to this author in setting personal guidelines for these interactions.

<u>Questions</u>. Costa divides question types into three categories: gathering and recalling information (input); making sense of gathered information (processing); and applying and evaluating actions in novel situations (output). The first type is the one most commonly used in the traditional classroom. It directs students to feed back information, to verify that they have stored and can recall that information handed out by the teacher. The teacher desiring to move to the facilitative classroom should ask questions that fit into the processing and output categories as well. In the processing category the teacher's "questions or statements should prompt students to draw relationships of cause and effect, to synthesize, to analyze, summarize, compare, contrast, or classify the data they have acquired or observed" (Costa, 1985b, p. 127). In the output category the teacher should invite the student to use the concepts they have constructed in new situations. Students responding to this type of teacher statement or question can be said to be: "applying a principle, imagining, planning, evaluating, judging, predicting, extrapolating, creating, forecasting, inventing, hypothesizing,

speculating, generalizing, model building, and designing" (Costa, 1985b, p. 128).

__Teacher Response__. The second major aspect of teacher verbal behavior is, of course, how the teacher responds to students' statements and questions. Teacher responses can be split into two main categories: those that close down students' thinking and those that open up students' thinking (Costa, 1985b, p. 131). Both criticism and praise can be viewed as teacher behaviors which will close down thinking (Costa, 1985b; Gordon, 1974). If criticism and praise are frequent teacher responses, the focus of the student will be on trying to avoid the former and gain the latter from the teacher rather than on the current learning task. The use of wait time and what Gordon calls passive and active listening responses are behaviors which will invite students to think autonomously (Gordon, 1974). Studies by Rowe indicate that the amount of time teachers wait after asking a question can have a significant impact on the type of responses given by students (Costa, 1985b, p. 133).

> Teachers who ask a question and then wait for a student's answer demonstrate that they not only expect an answer but also that they have faith in the student's ability to answer given enough time. Teachers who ask a question, wait only a short time, and then give the answer, call on another student, or give a hint, demonstrate their belief that the student really can't answer the question and is considered too poor a student to offer an answer or reason independently. (Costa, 1985b, p. 133)

Developing Interpersonal Skills. The previous discussion has dealt with verbal behaviors a teacher needs to engage in to question and respond to students. The model presented by Costa sets forth guidelines for the kind of climate needed for a facilitative classroom. An exceedingly powerful way of helping teachers develop the skills necessary to create this climate is set forth in the training program established as Teacher Effectiveness Training, a book and course developed by Thomas Gordon (1974). A major premise for this training and an important concept to be aware of in creating a facilitative classroom is that no learning can take place unless the teacher has been able to establish good, intimate, interpersonal relationships with his/her students. "Students are freed to learn only when the teacher-student relationship is good" (Gordon, 1974, p. 24).

> The relationship between a teacher and a student is good when it has (1) Openness or Transparency, so each is able to risk directness and honesty with the other; (2) Caring, when each knows that he is valued by the other; (3) Interdependence (as opposed to dependency) of one on the other; (4) Separateness, to allow each to grow and develop his uniqueness, creativity, and individuality; (5) Mutual Needs Meeting, so that neither's needs are met at the expense of the other's needs. (Gordon, 1974, p. 24)

Through the development of the skills of passive and active listening, sending "I-messages" about personal needs, and engaging students in the problem solving process to resolve conflicts of needs, the TET framework offers teachers a powerful, explicit way to develop the interpersonal skills necessary to create the climate of a facilitative classroom (Gordon, 1974).

Many of the important components necessary to create a facilitative classroom are summed up nicely in the following quote from Everybody Counts:

> Teachers' roles should include those of consultant, moderator, and interlocutor, not just presenter and authority. Classroom activities must encourage students to express their approaches, both orally and in writing. Students must engage mathematics as a human activity; they must learn to work cooperatively in small teams to solve problems as well as to argue convincingly for their approach amid conflicting ideas and strategies. (National Research Council, 1989, p. 61)

CHAPTER III

ONE TEACHERS'S STRATEGY TO EFFECT CHANGE

Establishing a Framework for Change

The Standards issued by the NCTM serve as a challenge to all of us engaged in mathematics education--a challenge to create new curricula, to write new textbooks, to develop entirely different means of assessment, and to create a new and different classroom experience. It is this last factor that the author working in a seventh grade classroom could most reasonably expect to effect. The following discussion considers the constraints on and the freedoms of the teacher in that classroom.

Constraints. First, there were the demands on time and structure over which the teacher had little, if any, immediate control. These included: responsibility for 90-100 students, 5 classes per day, 3 levels of instruction, 45 minute periods, two 45 minute periods per day for preparation (one being during a supervised study period), after-school responsibilities (3 to 4 days per week), an A to F marking system, grades to be generated for report cards issued four times per year, and a 180 day school year.

Second, there were the issues of curriculum to be considered. The teacher in the classroom faced 180 days of lessons to generate. Since the textbooks did not offer lessons which fit the new model, the teacher would have to search out sources of creative lessons or develop her own.

The third thing to consider was the group of students entering the classroom. They came from three different elementary schools with a great deal of anxiety about the transition to the Junior High School. The demands of changing classes, entering a new school, meeting new peers, and working out relationships with five or six new teachers with higher demands and expectations than those experienced in elementary school put a great burden on many of these students. For most of them mathematics had previously been taught in the traditional way. Combine that with the onset of puberty and all of its turmoil and the stress and strain on the student can be seen as a significant concern in initiating change.

The fourth set of factors were the author's own concerns about initiating any changes. These included: a recognition of the seventh grade classroom as part of a continuum of experiences the students have with mathematics which must be coordinated with what has come before and what will come after; a sense that whatever experiments the author tried, these students would only have one crack at seventh grade and a desire to at least do them no harm; and a recognition that the changes required of the teacher were quite extensive, that the new teaching style was a great departure from that which was familiar and comfortable, and that there was no natural support system in place to aid in making these changes.

Freedoms. The teachers in the author's school had a great deal of autonomy in experimenting with and determining curriculum. In general, this curriculum had been determined by the topics listed in whatever textbook was currently being used. Coordination was informal and was dependent on the personal relationships that existed among the teachers in the school. Articulation among different grades was also somewhat informal and was based on irregularly scheduled annual meetings with staff from the elementary schools or high school.

Junior high teachers selected their own books and curricula materials. Any major changes in the status quo would necessitate explanation and justification to the school administration, school committee and parents, especially if such change required any significant monetary expenditure. However, in general, there was a fair degree of professional respect afforded the teacher with freedom given for experimentation.

A Plan for Change

In thinking through possible changes which could be made in the classroom within the framework established above, the author decided it was unreasonable to expect that she and her students could move into the facilitative classroom all at once. The goal was to start to move the classroom into a new direction, a direction only partially clear to the teacher and exceedingly different from what she and her 90 seventh graders had experienced before. The author had long believed that if mathematical skills could not be used by students to solve problems then the acquisition of those skills was meaningless. Taking a cue from a statement reiterated in many places in the NCTM Standards that "problem solving should be the central focus of the mathematics curriculum" (NCTM, 1989, p. 23), the author decided to set aside one day a week to engage students in problem solving experiences.

Discussion in the first two chapters has emphasized that students in the traditional classroom experience mathematics as a series of independent skills with often little sense of when these skills might be personally useful or applicable. Yet the demands of our society necessitate that we train our students to be effective problem solvers. This cannot happen unless they have been exposed to and coached through many and varied problem solving experiences. The time set aside on problem solving day would be used to engage students in explicitly working on problem solving. The initial goals set forth were:

-to give students experience in solving non-ordinary problems of all sorts,

-to model effective problem solving,

-to explore the problem solving process itself,

-to identify and develop some explicit problem solving strategies,

-to provide the experience of applying mathematical and thinking skills,

-to reflect on the process,

-to communicate and reason verbally and in writing about mathematical ideas,

-to learn to work in cooperative groups.

<u>Guidelines for Initiating Change</u>. The author developed the following guidelines as the year began:

1. Introduce the students to the idea that one of the main reasons to study mathematics is to help them learn to solve problems. Delineate the difference between an exercise and a problem. Exercises help develop skills, like multiplication and changing fractions to percent form. Much of what students experience as "problems" are

merely exercises. "Whenever there is a gap between where you are now and where you want to be, and you don't know how to find a way to cross that gap, you have a problem." (Hayes, 1989, p. xii)

2. Help students realize they already have some problem solving ability, but that that they could learn to be more effective problem solvers. To do this: develop and use a problem solving plan; identify and consciously use beneficial problem solving strategies.

3. Expose students to experiences which would enlarge their view of what mathematics is.

4. Involve students in sharing ideas, posing questions, reasoning about mathematical ideas.

5. Look for ways to change the teacher's role from that of director and authority to that of coach, monitor, facilitator.

6. Search for problems, units, ideas developed by others to experiment with and use in the classroom.

<u>Lessons which Fit the Model</u>. With these objectives in mind the author began problem solving every Thursday. In making plans for these Thursday lessons, the author, in addition to developing original curriculum engaged in a search of activities and lessons already developed by others. The types of activities searched for were:

1. Lessons which included problems which:

-were open ended

-had many possible answers or many solution strategies

-required divergent thinking or had many steps.

2. Lessons that provided opportunities for students to apply and/or extend skills and concepts.

3. Lessons where solutions or solution strategies were not readily apparent.

4. Lessons which incorporated working in cooperative groups and the use of manipulative materials.

The next chapter will discuss lessons outlined in a videotape series developed by Marilyn Burns (1989a,b,c) that the author selected as being representative of the facilitative model of mathematics instruction. Justification of their selection is also included.

CHAPTER IV

THE LESSONS

EXPLORING THE RELATIONSHIP BETWEEN AREA AND PERIMETER

In her videotape series, Mathematics: for middle school, Marilyn Burns provides a model of teaching that is analogous to the model for a facilitative classroom established in Chapter II. The series is designed to be a resource in training middle school teachers to plan and facilitate problem solving lessons for sixth, seventh, and eighth grade students.

> The tapes show teachers how mathematics classes look when students engage in problem-solving activities, work cooperatively in small groups, use manipulative materials to help develop understanding, and present their ideas in whole class discussions. The videotapes also give teachers guidance for responding to students in problem-solving lessons that focus on thinking and reasoning. (1989a, p. 5)

The lessons presented in the second tape of the series focus on modelling the teacher's role in creating classes as described above. The author selected four lessons, three from this second tape and one from the first, to use as the basis for a unit exploring the relationship between area and perimeter and exemplifying the model established in Chapter II.

The essential characteristics of the lessons stated in the introduction to the tape series reveal these connections:

1. Students are presented with problems to solve that encourage their mathematical thinking and reasoning.

2. Emphasis is placed on having students explain their thinking, both verbally and in writing.

3. Students work cooperatively in small groups to maximize opportunities for explaining their ideas.

4. Manipulative materials are used whenever possible to help students bring meaning to abstract ideas.

5. Different areas of the math curriculum are integrated rather than isolated by specific skills or topics.

6. Students' misconceptions and errors are seen as indicators of confusion or partial understanding of math ideas and are accepted as natural to the process of learning.

7. Concepts are presented in a variety of ways and, as much as possible, are embedded in contextual settings.

8. Rather than explaining a new idea that the students then practice, lessons first engage children in problem-solving activities from which understanding of math concepts can emerge.

9. Calculators are available to students in all lessons and homework assignments.

10. Homework is used to further students' understanding and problem-solving abilities, rather than to practice skills. (1989a, p. 6)

Description of Lessons

Following is a description of the four lessons selected from Marilyn Burns' videotape series, together with additional commentary and interpretations made by the author.

Lesson 1: The Area Stays the Same (Burns, 1989b). (Requires 2-3 class periods). This lesson requires the students to gather into their cooperative groups. Each group is given a 5" by 8" index card, a ruler, a pair of scissors, a large sheet of paper, and a meter stick. The teacher directs them to draw and cut out six squares from the index card. Each square is 5 cm. on a side. The perimeter and area of a 5 cm. square (20 cm. and 25 square cm., respectively) is elicited from the students and recorded on the board. The students are asked to make different shapes from five of the squares, leaving the sixth one intact. They are to do this by cutting up each of the five squares and retaping the pieces together in a new configuration. The cut-up pieces must be retaped so that the resulting shape has no interior holes but rather forms a contiguous region. The large sheet of paper is to be used to make a chart of perimeters of the uncut square and of the five retaped shapes (see Figure 1).

The teacher discusses with the class the fact that the area of all of the shapes will be the same. When all groups have completed their task and taped the shapes under the perimeter lines on their charts, they are directed to look at the resulting chart and write down any general statements they can make about the relationships between the shapes and the lengths of the perimeters.

After all groups complete this task, they are asked to share their statements with the class.

This much of the lesson is clearly modelled on the videotape (Burns, 1989e) as discussed in the accompanying guide (Burns, 1989b). An additional statement is made to put all of the general statements on one sheet, duplicate them so that copies can be given to all of the groups for general group discusion about their truth or falseness.

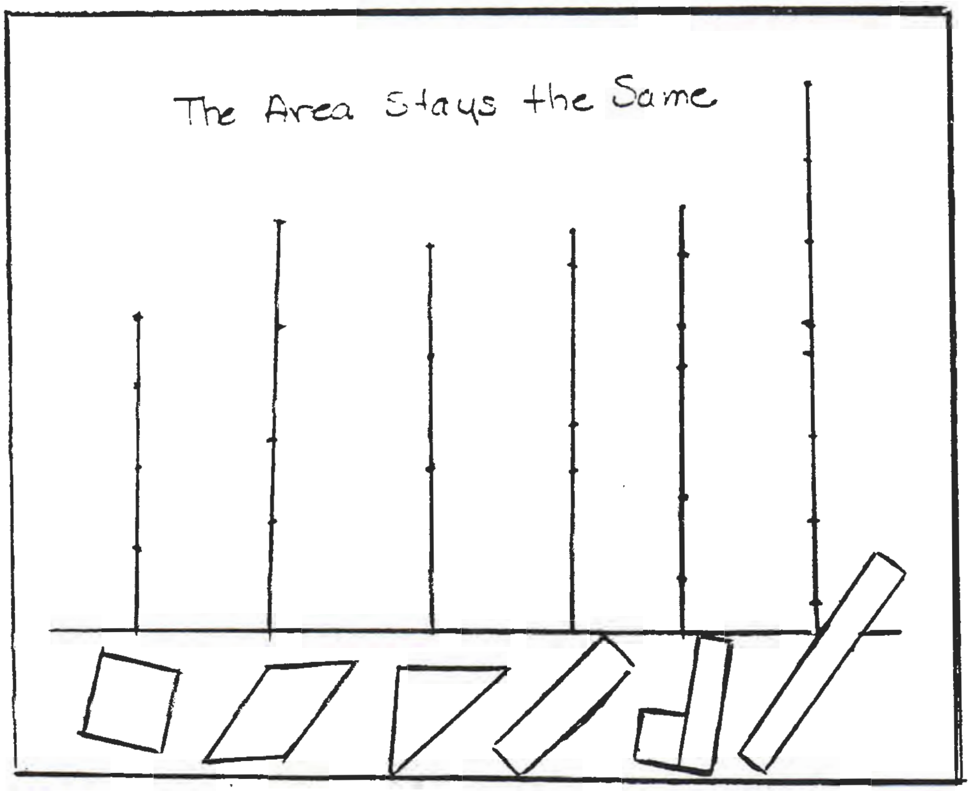

Figure 1. Chart of Perimeters

Homework assignment for further exploration: Each student is given worksheet 1 (see Figure 2). This worksheet asks the student to draw three more shapes keeping the perimeter the same 30 cm.

When this author implemented the lesson, she chose to collect all of the statements, xerox them and hand them out to the groups for discussion on a subsequent day. The groups were asked to review all of the statements and classify them into three categories: 1. those they agreed with; 2. those they believed were incorrect; and 3. those about which they could not decide without further exploration. During this session all of the charts were displayed. This was followed by a whole class discussion with statements being put into the three categories on the board.

This is an example of engaging students in a metacognitive activity. They were asked to reflect on the observations made on a previous day. Class discussion included delineating reasons for their choices.

Lesson 2: The Perimeter Stays the Same (Burns, 1989b). (Requires 2, 45 min. periods). This lesson begins with the teacher asking the students in their groups to check each other's worksheets for correctness of perimeter (30 cm.) and correctness of area (varying). When the students have completed this task, they are instructed to look at

THE PERIMETER STAYS THE SAME

Name ______________________________

Draw 3 <u>different</u> shapes on the squared centimeter grid below.

Follow these rules:
(1) Stay on the lines.
(2) You must be able to cut out your shape and have it all in one piece.
(3) Each shape must have a <u>perimeter</u> of 30 cm.

Record the <u>area</u> inside each shape.

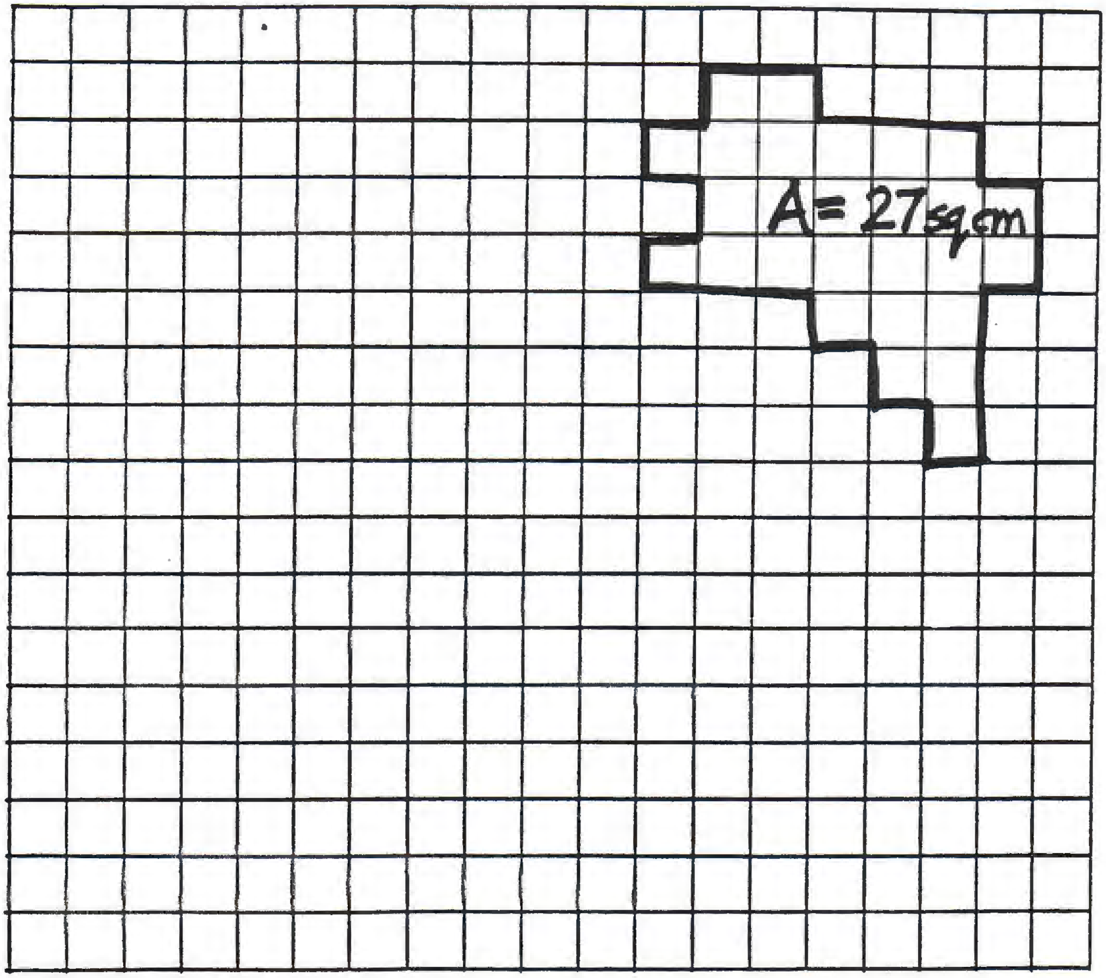

Figure 2. Worksheet 1: Assigned for
Perimeter Stays the Same
Adapted from Burns, 1989b

the set of all shapes from their group and cut out the one with the least area and the one with the greatest area. Each group then tapes these shapes on two pieces of construction paper hung at the front of the room -- one for least area and one for greatest area. The class is asked to make any observations they might have about the two groups of shapes. The students are then directed to compare and contrast the two groups of shapes and write down general statements about the similarities among shapes in the same group and differences in shapes between the two groups.

Homework assigned: Worksheet 2 (see Figure 3) is handed out to be completed before the next class. Each student is instructed to come up with an estimate of the area of the hand.

During implementation of this lesson, the author chose to hand out xeroxed copies of all of the statements and asked the groups to choose ten they thought were correct. They were directed to reflect on the previous lesson for ideas. Again, this sort of lesson lends itself to engaging students in metacognitive processes. They are asked to reflect back on previous thinking.

<u>Lesson 3: The Hand Problem</u> (Burns, 1989b). In this lesson, students are first asked to discuss the homework worksheet (Figure 3) with a partner. They are to compare

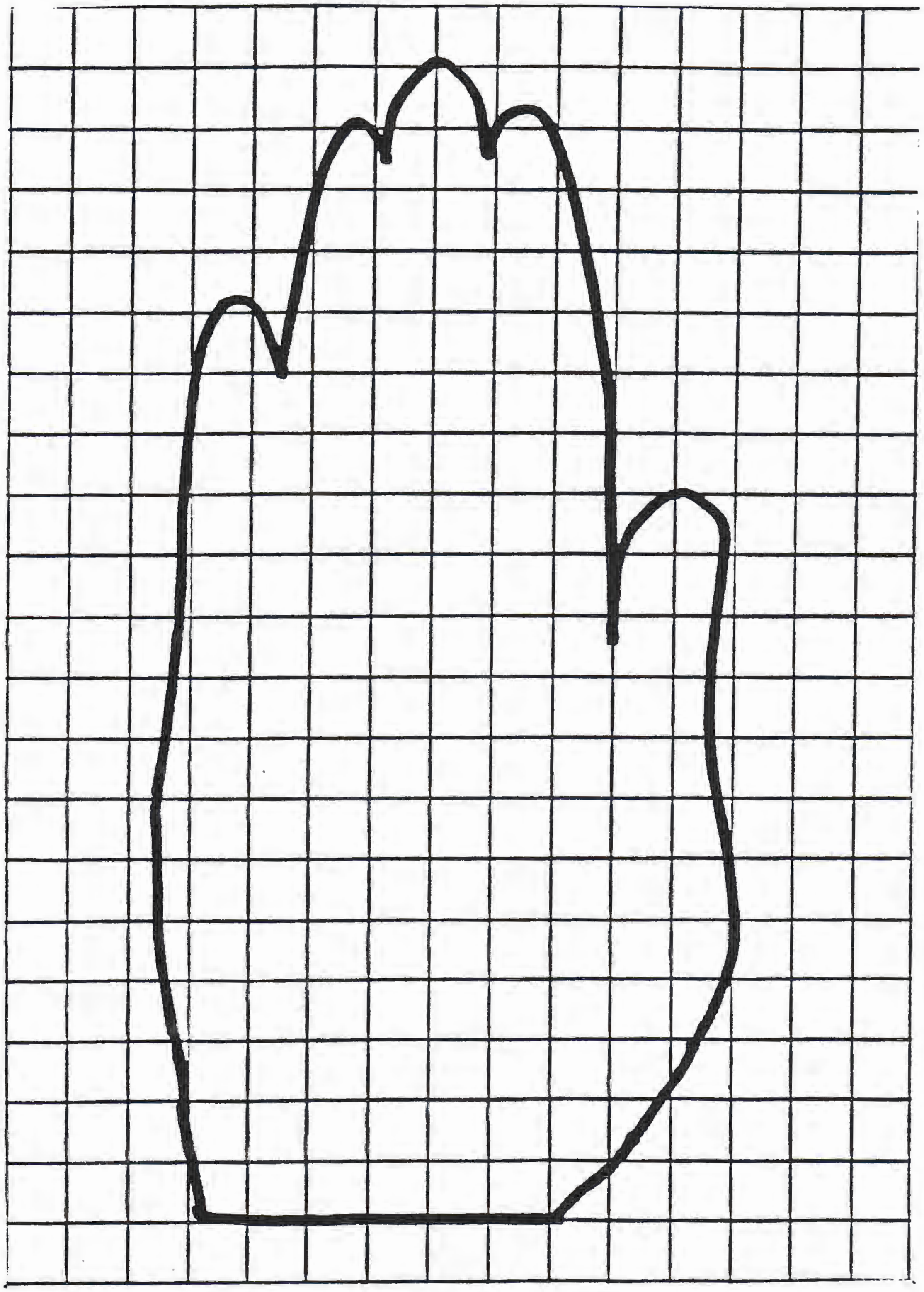

Figure 3. Hand from worksheet 2: Assigned for
Area of the Hand
Adapted from Burns, 1989b

results and the method each used. They then assemble into groups of three or four students and do the same sharing in the larger group. Next the teacher engages the class in a whole group discussion. The imprecision of all measurement is discussed. Students are asked to share the different methods they used for this task and the idea is reinforced that there is not "one right way."

An addition to this lesson made by the author is to record the range of answers on the board. It is then easy to observe that there is a general clustering of answers around some central value, so that it is likely that the real area is equal to that central value plus or minus a few square centimeters in either direction.

Next, the teacher introduces to the students a method of calculating the area of the hand which was suggested by "one of my students." The method is not a correct shortcut and the purpose is to see if the students' previous experiences have led them to perceive the conceptual error inherent in the method. First the students are asked to respond to the method. Then they are asked to work in their groups to explore the idea further and to come to a consensus about whether the method is a good idea and why.

The method presented to them and which they are asked to explore with string, scissors, and tape is to cut a piece of string which is exactly the perimeter of the hand; then, form a square on the centimeter paper from the cut out piece of string (see Figure 4). The proposition is that the area of the square will be equal to the area of the hand and it will be much easier to calculate the area of the square.

The groups are asked to share their ideas after completion of this activity in a whole class discussion.

<u>Lesson 4: Make the Perimeter Larger</u> (Burns, 1989a). This lesson centers around an exploration with colored tiles. The students gather in their groups and are given containers filled with two different colored tiles. Initially each student is given an activity to work on individually. The activity is to first build a displayed shape with eight tiles of one color and then record the area and the perimeter of the constructed shape.

Next each student is instructed to add tiles of the second color to his/her shape until the shape has a perimeter of 20 units. At this point all students are

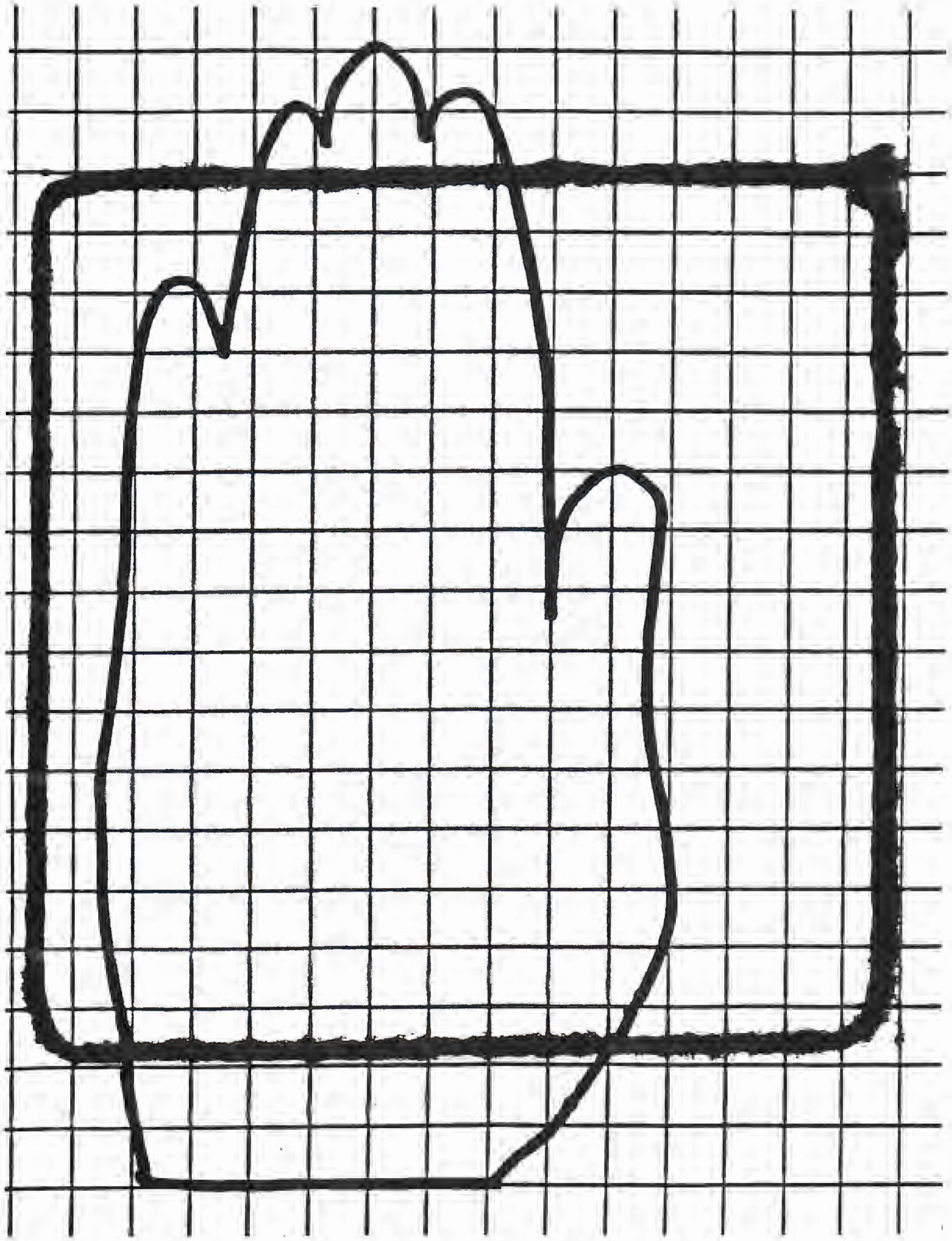

Figure 4. Hand from worksheet 2 with string square of equal perimeter superimposed.

instructed to look at the shapes of the other students in their groups and verify area and perimeter of the new shapes. Each student then records on graph paper everyone's solution.

Next, as a group, the students are directed to discuss and explore how the perimeter changes when they add a tile to the shape. They are to record the results of their discussion.

Discussion of Author's Modifications

The issue of homework continues to be a controversial one. As cited above Burns feels that homework should be used to further understanding. The author still believes that homework is also useful for helping students develop skills. In planning this unit for use in her classroom, the author decided to add additional homework assignments to those given by Burns. These assignments are presented in the Appendix and were given out after lessons 3 and 4. They were designed to give the students more experience working with the concepts of area and perimeter and to explore the relationship among various units of area. In addition, it was the author's intention at the end of the series of lessons to ask students to compare these lessons with more traditional mathematics experiences. By assigning homework on a regular basis during these lessons, the author hoped to counter any preference for the type of lesson presented in this unit as being attributable to their delight at being free of homework.

Burns' Support of the New Model

In addition to the modelling of lessons on the videotapes along with the accompanying discussions in the booklets which come with each tape, Burns appears on the videotapes to share her philosophy of middle school mathematics education. In tape two (Burns, 1989e) she indicates her belief that the traditional model of teacher presentation followed by practice should be reversed. Students should be engaged in mathematical activity and the teacher's role then shifts to that of aiding the students to draw out the mathematics from these experiences. The teacher becomes the facilitator in helping kids construct their own mathematics.

In further discussion of the teacher's role, Burns shares her belief that this does not mean leaping in and rescuing students when they are not understanding. Confusion is a natural part of the learning process. A teacher's role is to uncover this confusion. This can be accomplished by the teacher asking for students' solutions and serving only as a recorder. When differences of opinion occur, often the teacher's best approach is to give the controversy back to the students as a problem to explore. The teacher can ask students to explain their reasoning behind any solution to either the class, a small group or a partner. If difficulties cannot be resolved in this way, then the teacher must decide what other learning experiences need to be offered (Burns, 1989e).

As this series of lessons and the approach used by Marilyn Burns to teach middle school mathematics seemed to this author to exhibit all of the characteristics outlined in the facilitative model and supported the constructivist view of learning, she chose to implement them in her classroom as part of the problem-solving curriculum.

CHAPTER V

LESSON IMPLEMENTATION

This chapter contains a discussion of the author's implementation in her classroom of the lessons described in the previous chapter.

The Mechanics

The decision was made to try the lessons with the three average level seventh grade classes the author instructs in a relatively affluent, suburban community. Class size ranged from 16 to 22 students. Since problem-solving was being conducted once a week on average, the decision was made to try these lessons within that framework. Thus, these lessons would be separated by the more traditional presentations and lessons that occurred on the other days of the week.

The students were informed that their teacher was going to try an exploratory series of lessons dealing with the relationship between the concepts of area and perimeter; that the lessons were an experiment their teacher was trying as part of her own graduate work; that the lessons would be videotaped so that their teacher would have a record of how the lessons went and not to find out who was "being bad." A videocamera was set up in the corner of the classroom to try to capture both the interaction among the students in one particular group within each class and the interaction between the teacher and students in whole class discussion in each of the classes.

The students had been working on problem-solving in cooperative groups for several months before these lessons were presented to them. Students had been assigned to groups of three or four students and were to remain in these groups for the entire series of lessons. These groups had been set up by the teacher to contain heterogeneity of ability and to have combinations of students who had not worked together in previous experiences. The students were familiar with and were used to functioning under the "Groups of Four Rules" which were posted in the classroom. These are:

1. You are responsible for your own behavior.
2. You must be willing to help anyone in your group who asks.
3. You may not ask the teacher for help unless all four of you have the same question.

(Meyer, 1983, p.5)

The groups moved their desks together at the start of each lesson so that they formed a "table." At the end of each class period the students moved the desks out of this arrangement, gathered and returned materials they had used, and stored the group-generated materials as directed by the teacher.

Lesson Management--Teacher as Coach

Before beginning the series of lessons the author set forth some personal guidelines for the role of teacher as facilitator. The lessons as designed and modelled in Marilyn Burns' tapes show the teacher setting up the activity by explaining what students are expected to do. Then, as noted in Chapter IV, the teacher is expected to monitor students' progress, give guidelines for group discussions, and act as moderator for whole class discussions. Although these discussions were modelled on the tapes, there is no certainty of predicting where the discussions will lead in any particular class. The facilitating guidelines used were as follows:

1. Move around and observe each group briefly. If the group seems actively engaged listen in for a few minutes without comment and then move on.

2. If a group seems in conflict or has asked for teacher input, seek some way to get them unstuck or resolve their issue in the least authoritative manner. This means the teacher should focus on asking students what David Perkins calls "prompting questions" (1986) to help students find their own solutions or answer their own questions. These kinds of questions help students to access their own "inert knowledge" (Bransford et al, 1986;

Perkins, 1986) and bring it to bear on the issue. The teacher should refrain from moving on to the more directive "hints" and most directive "provides" as suggested by Perkins unless the students' response indicates a total inability to move forward on their own (1986). Examples of prompts are: Read the problem. Read it aloud. How would you explain the problem? What might you do next? What strategies might you try?

3. If observations indicate a general class confusion, difficulty, or question, interrupt for class clarification.

4. When groups are involved in problem-solving, ask monitoring questions. Schoenfeld (1985a) has stated that the teacher's role in doing this is to model for students what ultimately should become internal monitoring behavior. Examples of this kind of question are: What are you doing? Why are you doing that? Why did you decide to use that strategy? These questions are asked whether the teacher observes the students proceeding in an appropriate or inappropriate manner as a means to get the students to confirm and communicate their thinking. Initially a student may perceive these kinds of questions as an indication that he/she is making an error (Schoenfeld, 1987; Kamii, 1985). The goal is for students to feel confident in the path they are taking or to know what and why they are currently doing what they are doing without

the need for external validation even if what they are doing is merely fishing for a possible solution path. In other words, the teacher is helping the student to think about his/her thinking; to develop metacognitive control (Schoenfeld, 1985a).

Lesson Outcomes--Difficulties and Successes

The following discussion will focus on some general observations made of the lesson outcomes with some specific examples taken from the videotapes recorded in the author's classroom.

Stage Setting Activity. As stated above, the teacher introduced the unit as an exploration of the relationship between area and perimeter and told the students they would be working on it for several consecutive Thursdays. The students were given an activity with colored tiles that allowed them to play and become comfortable with the materials. Students of this age like to play with tiles, build towers, topple rows using the domino effect, etc. Time was set aside initially to help them become comfortable with the tiles and to reintroduce the notion of area as a tangible covering of a surface with tiles. One tile serves as a concrete model of a unit of area measurement. The students were asked, as part of their play, to form the tiles into different sized rectangles and squares. The teacher used the opportunity to explore, by conversations with individual groups, whether students knew what rectangles and squares were and that a square is a special kind of rectangle. An overhead was then posted bearing problems for exploration in their groups. Students were asked to record the groups' results for further exploration on a homework worksheet.

In general, the author thinks that some sort of stage setting activity and discussion is necessary when introducing a series of lessons such as these. The students' thinking (in this case, on area and perimeter) can be explored by worksheet and/or class discussion. Some mutual agreement should be made about definitions and terminology to be used. In this case the author led each class in a discussion of the students' understanding of the meaning of area and perimeter as well as the units used for each measurement. The discussion concluded with the establishment of working definitions for the two measurements. The tiles served as a useful mechanism to make the definitions arrived at tangible and concrete.

Teacher's Roles. In evaluating her own performance on managing the lessons, the author kept in mind four different functions of the teacher as facilitator: teacher as coach, teacher as model, teacher as monitor and observer, and teacher as director. Initially in outlining any activity, the teacher appropriately is the director. The teacher needs to give clear directions for the activity and then, once the activity starts, monitor and observe whether the directions have been clearly understood and redirect groups that have misunderstood. This monitoring and redirecting serves as one example of the teacher acting as a coach.

The author did note when viewing herself on the videotapes that at times she was very directive when it might have been better if she merely observed and then used the information gained from the observations for feedback or to determine future experiences. As an example, when noting that only one person was doing all the work in a group, she said, "There's only one person working in this group; take turns." An alternative way to deal with this is to set up time at the end of the group work for groups to evaluate themselves, with other time set aside to discuss effective group skills. A statement such as: I like to have everyone's attention when I give directions so I don't have to repeat myself many times and waste my time (Gordon, 1974) might have been given in place of the statement on the videotape: "Eric, Jan, etc., you're not paying attention." The author believes that a teacher should extend invitations to students to pay attention and to be considerate of others. If the teacher presents reasons for holding these values, student compliance may be more likely than if they are imposed as rules. The primary focus is in establishing a risk-free, comfortable classroom where all ideas can be expressed freely and conflicts expressed and resolved without fear of being put down or embarrassed.

Managing class discussions of the students' observations is a clear example of the teacher functioning in the facilitative role. It is in this kind of activity that the traditional mathematics teacher may find the most difficulty determining and maintaining the proper role. The tapes reveal the author's behavior as still somewhat "guiding" to "the right answers."

Burns' lessons sometimes call for students to share their initial reactions to an idea before they have had a chance to confer with other students. The author found that students were often uncomfortable sharing observations with the whole class. There would be little or no response from the students when they were asked to give their reactions and a deadly silence would settle over the class. The teacher needs to be conscious of the value of wait time (Costa,1985b), p. 133) in this sort of situation. Sometimes if the questions were rephrased, a few students would venture an opinion. Even if only a few ideas are put forth the author believes this sort of initial discussion is useful to help focus the subsequent discussions in their small groups. Also the more times the class engages in these discussions with all ideas being accepted, the more forthcoming students will hopefully become.

Acceptance Without Judgment. The author concurs with the idea that acceptance of students' ideas in a non-judgmental way should aid students in becoming more self-reliant and less approval seeking. Both the author and her students found this mode of interaction unfamiliar and difficult at times. This became apparent to the author when viewing the videotape of the lesson where students compared the result each got for calculating The Area of the Hand and the method each used (assigned for homework prior to the lesson of the same name). The author asked one group to explain their answers, which ranged from 125 sq. cm. to 137 sq. cm. and for the methods each used. Although she felt that one of the student's methods was clearly better than the others, she made no comment about whether one method was better than another. After she left the group, it was apparent from their conversation recorded on the videotape that the students were expecting some additional response from the teacher, some conclusion about which was the best method, who had "the correct answer." The students definitely were waiting for some external judgment to be made to validate or invalidate their own thinking. Praise as well as disapproval of a student's method is, of course, also a judgment. If the goal is to help students become independent in their thinking, striving toward a posture of non-judgmental acceptance becomes extremely important. Being aware of this in student-teacher interaction is not always easy for this author, but she hopes to help students recognize that, although some methods are more efficient than others, problems can be solved by many different, equally valid methods.

Directing Discovery. It is sometimes not clear to the teacher what directions might yield the most interesting results or how specific to be in directing students as to what they are looking for. In reflecting on this, the author has come to believe it is better to start with a somewhat general direction and to get more specific only if a group (or the whole class) seems to be really stuck. An example of this uncertainty and some ideas about response to it arose during the lesson, The Area Stays the Same. Burns' directions only stated that students were to make general statements about what they noticed. The purpose is to direct students to observe what characteristics of a shape affect the perimeter of the shape. When students seem confused about what they are expected to do, the author suggested they ask themselves questions such as, "What about a shape makes the perimeter shorter? What makes it longer?" "If you can see any sort of pattern, write down what you see."

Although, in planning any lesson of this sort, the teacher aims to help students uncover certain specific relationships, the teacher should also be open to a student observing something new, not anticipated by the teacher (again avoiding that search for the "right answer"). The goal here is to enable students to feel comfortable in doing their own thinking, their own observing and for the teacher to support that.

Helping Students Focus on Ideas. Several times in these lessons the students were asked to write general statements about things they had observed for later sharing with the class. The statements were collected and xeroxed for group discussion the next day. The author elected to give points for group completion of a task so students' names were put on each groups' statements. In the first xeroxing, the students' names were still attached to the ideas. After observation of students' behavior, the author determined that the names should be removed for classroom distribution.

Students benefit by being recognized publicly for production of exemplary pieces of work. Also, students of this age can often be drawn into an activity by seeing their names and their friends' names attached to the work in some way (on student-created worksheets, for example). In this activity, however, with the names still attached students reacted more to the personalities of the students connected to the ideas than to the ideas themselves. The students spent time and energy searching for their group's and their friends' names but comments overheard by the teacher indicate this interest was counterproductive for this particular activity. "Let's keep statement number eleven because that was Eric's." "Of course, we'll keep all of OUR statements." This activity was more effective when the students approached the ideas without names attached. More reflective thought emerged.

Additional editing of the general statements should focus on making sure there is a variety of ideas for the students to consider. The teacher can add some ideas from another class that no one thought of in the current class, and eliminate some repetition. In general it is desirable for the class to have between 15 and 25 statements to consider. If there are too many statements the groups will bog down from having too much information to process.

Question Posing. These lessons seem to lend themselves naturally to the students and the teacher doing some question posing of their own. It turned out to be a rather exciting outcome. Often students would get so involved in exploring the answer to a question that had arisen from the class or small group discussion that the teacher would have a hard time getting them to stop to clean up for the end of the period.

In the lesson <u>The Perimeter Stays the Same</u> the questions that seemed to arise naturally when looking at the shapes grouped by least and greatest areas were: What is the least area we could make keeping the perimeter the same? the greatest? Have we found them already? How could we be sure we have them? What if we remove the restriction that we stay on the lines and form a rectangle only 1/2 a tile wide? What IS the area of a rectangle 1/2 tile by 10 tiles long? These questions can lead to the clarification and development of other mathematical concepts. Once the students start dealing with 1/2 tile widths, fractional multiplication emerges naturally.

The teacher needs to be alert for these sorts of possibilities and be flexible in allowing students to explore.

<u>Dealing With Student Misbehavior</u>. Of particular concern to the author upon reviewing the videotapes was the off task and uncooperative behavior of several of the students. In one group made up of four students, two of

them engaged in talking about personal issues while the other two did all of the work required. In another group, one student spent the time teasing and taunting other group members and generally engaging in behavior that hindered the work of the group. In a third group, one student took charge of most of the materials and control of all of the work rather than working cooperatively with the other two non-assertive group members. Students' comments from these groups indicated that many students were unhappy working with others who inhibited the group's work and didn't quite know how to deal with it. This pointed out to the author the real need for the creation of positive interdependence among group members and for finding ways to make the students individually accountable.

Johnson et al (1988) state that this is the essence of helping students to work in cooperative groups. Students must be taught interpersonal and small group skills explicitly and must be given time and procedures for assessing how well the groups are functioning. Teaching skills, again, is in contrast to the teacher trying to modify behavior by pointing out the bad behavior to the student involved. The author, implementing cooperative groups this year for the first time, did not allow enough time for students to process their own group's functioning on a consistent basis. With the time pressures of the 45 minute period this part often got left out. It was difficult for the author not to indicate in a judgmental, accusatory way dissatisfaction with students who were not on task or were teasing other students. The methods recommended by Johnson et al (1988) and Gordon (1974) for placing the responsibility for their behavior onto the students and allowing for self and peer evaluation are surely worth pursuing by any teacher as a way of handling this conflict between teacher and student.

An additional source of off task behavior is discussed in the following section.

<u>Constructions Not Made</u>. The difficulty of pressing students to make connections and construct their own meanings on a time schedule became apparent when reviewing the videotapes made in the author's classroom. In the second part of lesson 3, which deals with the area of a hand drawn on a worksheet and given out as a homework assignment, the students engaged in a discussion of the different methods used to find the area of the hand. The author recorded on the board the range of values obtained and stressed the idea that the area was actually somewhere around 130 sq. cm.. The students were then asked to consider an easier, "student suggested" method for calculating the hand area (see <u>Lesson 3: The Hand Problem</u> described in Chapter IV). When first planning and

thinking about presenting this lesson, the author thought most of the students would easily recognize that the method would not work. The previous two lessons should have revealed to them that the area of two different shapes with the same perimeter may be very different, that perimeter is not a determining factor for the area of a shape. Students seemed conscious of this but when actually asked whether the method would or would not work very few actually said it would not.

After performing the experiment with string and tape and discovering that the square obtained has an area somewhere between 196 and 225 sq. cm. (see Figure 4 in Chapter IV), the students, for the most part, still did not make the connection that the area difference between this and their homework calculations was due to this lack of connection between area and perimeter. Many students in their statements thought the discrepancy arose due to the difficulty in measuring the perimeter accurately; their thinking remained stuck at the mechanical level. Even if the connection was made it was not held strongly as the following samples of students' statements from three different groups show:

1. "It is not a good idea because the answer was way to heigh than the average we discussed in class. The string cones out to big because when you go over the finger so you go over the same place twice."

2. "This method is good because it makes the task simpler. However, it is not easy to line up the string to measure it."

3. "No." [It is not a good method because] "you can make different rectangles and they all calculate differently. It can slip and you would mess up."

In reviewing the whole class discussion after their experiment, the author's frustration was plainly visible. She verbally told the students that, of course, the method was not a good one due to the fact that different shapes with the same perimeter do not usually have the same area, that perimeter is not a determiner of the area of a shape as they have seen from their previous examples. The students nod their heads, but the videotapes reveal that although the students were aware of that fact, they did not really understand why that is the invalidating factor in the failure of the method. Their search for the reason for the method's failure was restricted, for the most part, to the difficulty they had in mechanically obtaining a physical perimeter by running a string around the edge of the hand.

Either the lesson should be presented after other experiences or more time and student discussion need to be given to enable students to make this abstract connection for themselves. An additional factor contributing to the difficulty of the lesson for the author's seventh graders was that this sort of requirement was not within their normal realm of experience. Seldom, if ever, had they been asked to decide for themselves whether some method presented to them is ineffective. The students, when asked to comment on the method, were plainly at a loss as to what to say. Their frustration was illustrated by some off task behavior: looking up in the air, humming, the recorder handing the paper and pencil for recording the group's ideas to other members of the group (and finding no takers), shoulder shrugging, etc. They were quite accustomed to accepting and working with methods in mathematics class which they did not fully understand; in other words just accepting the authority of any method presented to them and assuming that any fault lay in their application of the method not in the method itself. The need for presentation of this type of lesson more frequently becomes apparent, but also that the teacher cannot expect that students not used to this sort of thought demand will respond immediately.

Students' View of the Lessons

Analysis of the students' responses when asked to comment on the lessons outlined in Chapter IV revealed several common themes.

Cooperative Groups. The students all responded positively to working in groups with peers.

"I liked that we worked with groups alot".

"My favorite part of these lessons was being able to get into our groups."

Their comments consistently revealed that this was much more fun and also much more productive than having to deal with problems alone. If the student found the lessons hard or the directions difficult to understand, the opportunity to clarify afforded by the freedom and safety of the group was much appreciated. Hearing different points of view, having mistakes corrected by peers were stated by many as positive experiences.

"The lessons were fun and interesting."

"I had some fun with the perimiter making weird shapes."

"What I liked about this lesson was that we did it in groups so if we misunderstood the directions or don't know what to do you have someone to ask."

"I think these lessons taught us how to work together as a group."

Several students did indicate frustration with group dynamics. Problems with being listened to by other group members, with getting group members to stay on task indicate the need to focus some time and attention on group and communication skills.

"The only thing I disliked was my group; a few people in my group don't want to cooperate all they want to do is talk about skateboarding, and stuff like that."

"Some of our group doesn't think very much."

Thinking. Students, in general, found these lessons more challenging, harder, requiring more thought but they indicated this was preferable to being passive recipients of information. Many stated that the lessons were not really mathematics at all because memorization and calculation skills weren't what was necessary, revealing a very narrow conceptualization about what mathematics entails.

"Regular math is memorizing."

"Experimenting and proving things yourself is better than 'recipe math.'"

"This kind of lesson made us think."

"We were forced to think and experiment with our ideas, which is good."

"We didn't have to calculate any hard things."

"In reg. math we take lots of notes and add, sub, div, and mult."

"I did always look forward to Thursdays."

"In other lessons, the teachers just tell us everything and it's harder to learn it when we actually didn't take part in experimenting."

Manipulatives. Most students found working with concrete objects enjoyable and helpful in understanding the concepts. Several students commented on the interaction with materials as a much better way to really understand what was going on; others just found it a pleasant change of pace from having to do "all that paperwork."

> "I did enjoy this project because we did it in groups and because...it was a 'hands on' experience."
>
> "What I liked about this lesson was some of the stuff we did we could do it phisically."
>
> "It's much more interesting and easier to learn when you can actually look and touch what you're doing."

Benefit to Teacher. One critical, important benefit to teachers that emerges from analysis of the implementation of these lessons is the importance of input from students as a guide to teachers. Having students work in groups and acting as an observer can help a teacher enormously in understanding where students are having difficulty. Asking students to reflect and give individual feedback on their experiences can also be of great help in enabling the teacher to learn and grow, to uncover problems. Just reading the reflections makes this author aware of the wonderful insights students have on what is happening and what might be beneficial in solving problems. The advantage to the teacher of this sharing can be significant.

Any mechanism that helps in this sharing will be beneficial. Having students keep journals which teachers look at and respond to, plus periodic requests for reflections on anything going on in the classroom can be of benefit. Learning occurs best in the context of human interaction and communication. Asking for and responding to students' reflections improves and opens up the contact between the students and the teacher. The teacher can expect that students of this age will be, for the most part, open and honest and, at times, brutally frank.

"I think you could have explained what we were supposed to do and how a little better."

"I thought that taking notes was boring."

"I think we should have tried to be more inventive, and tried making lessons that differed from the lessons shown on the 'Mathematics for Middle School' tape. It seems that almost everything we did duplicated the tape." [The author showed one of Burns' videotape to two of her classes.]

"Some lessons were interesting but some were very dull."

"I liked the way Mrs M. [the author] did it. [but] Yes, you could have changed your teaching methods. I noticed you taped the same group all of the time [therefore] you got only one way of solving problems."

"I thought math this year in 7th grade has been a great accomplishment for me considering I've never done great in math, but this year it has changed. Everything we have done has been understandable to me."

"Having a teacher be more enthusiastic encourages you to be that way also (most of the time)."

Conclusion

The strength of these lessons is best revealed by focussing on the amount and kind of interaction that went on in the author's classroom during their implementation. Rather than the teacher doing all of the talking and explaining and asking questions to which one or two students offered the correct response, the students became engaged in conversations about mathematical concepts. The teacher had the opportunity to hear what students were really thinking, to uncover student misconceptions. Opportunities were afforded for students to raise questions of their own for further investigation. All students had the opportunity to present, elaborate, and realign their ideas not just the student who had already grasped the concept the teacher was trying to teach.

Students of this age are enthusiastic, volatile, and active. The struggle for any middle school teacher is to engage students in such a way that their enthusiasm is applied in an educationally planned activity. The issue raised earlier of accepting students' answers without judgment can be extended to accepting students of this age where and for what they are without judgment. This can be exceedingly difficult for any adult. The middle school student's behavior often seems rude, unruly, and utterly incomprehensible. Working with students of this age must start with a basic acceptance of this behavior as being normal for the age. Marilyn Burns' lessons and their approach to students' learning of mathematics should be engaging to most middle school students at all levels of ability.

> As the students became interested in the activity and felt more confident with the work presented, they became clearer about their responsibility. They settled down and became more purposeful toward their work. (Burns, 1990, p. 22)

The author would like to stress the increase in confidence indicated in the above quote. The students in the author's seventh grade classroom have been accustomed to looking to external authority to validate how they are doing in mathematics. "Is that the right answer?" "Did I do that right?" They view mathematics as a subject for which they need this external support. They are accustomed to it not making sense, to using methods which they don't understand to get answers to questions which will be on the test, to not being able to rely on themselves. These lessons are designed to help students make their own connections when and if they are ready. It will take many experiences of the sort exemplified by the lessons presented in this thesis to help these students to come to rely on their own ability to make sense out of mathematics.

CHAPTER VI

REFLECTIONS

Polya, in his delineation of the steps in the problem solving process, labels the last step that of "Looking Back" (Polya, 1957). This chapter will fulfill this function by addressing the issues raised by the author's attempt to move into the facilitative classroom. The original problem defined in this thesis dealt with a perceived inadequacy in the mathematical educational system to help students become "numerate" in the Information Age. Even more specifically it involved a focus on the impact of the solution to this problem within the province of the classroom. Ultimately this must be the arena where any solution is played out. Any solution must deal with changes in what goes on in individual classrooms everywhere.

The author will move from consideration of the personal effects of the decision to make one individual classroom more facilitative to a more general consideration of the factors affecting all classrooms. In the ideal classroom all students are challenged to stretch and grow, are treated with individual respect, and have their creativity encouraged and not stifled. This atmosphere of a "risk-free" classroom can only be achieved by the teacher in charge of the classroom. The philosophy and the skills of the teacher must, of necessity, affect this outcome.

Reflective Learner

For this author, there has been a growing recognition that the teacher's posture must be one of teacher as reflective learner. The classroom must be the kind of place that encourages and nurtures the growth of all of its members, teacher included. The traditional teacher in struggling to change the classroom from authoritarian to facilitative must undergo a transformation. The teacher is learning a new role, but that new role does not include becoming an authority in a *different* way. The shift is rather *from* being an authority *to* being a reflective learner. As such, the teacher must be as supportive of the student within the self as of the other students within the classroom. In fact, unless the teacher can be forgiving of imperfection, of mistakes in the self, then there is no way that the teacher can offer these supports to others.

The difficulties AND benefits of this major internal shift are well expressed by Donald Schon in his discussion of changing from the authoritative professional to the "Reflective Practitioner":

> Whether the professional occupies a position of initial strength or weakness, the reflective contract

> calls for competencies which are strange to him. Whereas he is ordinarily expected to keep his expertise private and mysterious, he is now expected to reflect publicly on his knowledge-in-practice, to make himself confrontable by his clients.
>
> As the professional moves toward new competencies, he gives up some familiar sources of satisfaction and opens himself up to new ones. He gives up the rewards of unquestioned authority, the freedom to practice without challenge to his competence, the comfort of relative invulnerability, the gratifications of deference. The new satisfactions open to him are largely those of discovery--about the meanings of his advice to his clients, about his knowledge-in-practice, and about himself. When a practioner becomes a researcher into his own practice, he engages in a continuing process of self-education. When practice is a repetitive administration of techniques to the same kinds of problems, the practitioner may look to leisure as a source of relief, or to early retirement; but when he functions as a researcher-in-practice, the practice itself is a source of renewal. The recognition of error, with its resulting uncertainty, can become a source of discovery rather than an occasion for self-defense. (1983, p. 299)

There have been many teachers and would be teachers who have become inspired by the vision of a facilitative classroom who have given up and left the profession. Kamii discusses teachers who have "fought the system for a few years" (1982, p. 10) and then left public education. She points out the difficulties of this change, both internal to the teacher and external (administration and the public):

> The most difficult part of constructivist teaching is that it requires change, not only of a teacher's method of teaching, but also of his entire way of thinking about himself. It is extremely hard for a teacher to stop being the all-powerful, all-knowing center of the classroom. It is likewise extremely hard for an administrator to stop being a factory manager who supervises the mere filling of bottles. I

> think, in fact, that for the great majority of teachers and administrators already on the job it is impossible to change because it is too threatening to give up adult power. (1982, p. 9)

Crosswhite also has spoken of the difficulty of creating this facilitative classroom in our schools as an ongoing one (1987, pp. 270, 271). He shares his experiences in 1961, when working under Harold Fawcett supervising student teachers at Ohio State University. Fawcett modelled the teaching of mathematics using methods quite similar to those discussed in the facilitative classroom defined within this thesis. The students were generally unsuccessful in putting Fawcett's model into practice in the regular public school system. The problem, according, to Crosswhite was not in the model of mathematics teaching, but rather that Fawcett's ideas had not been adequately translated "into procedures that could reasonably be employed by a teacher working under the conditions that existed in the schools" (1987, p. 271). Crosswhite does not despair of this translation being made, but rather holds that it is an exceedingly difficult one and that the bridge that must be created across the gap between theory and practice must be built FOR the teacher in the classroom.

The author includes the preceding discussion to indicate that the task set, of changing the mathematics classroom, is not at all an easy one; setbacks and frustrations need to be recognized as inevitable; and the process will occur only gradually and over time. Both Kamii and Crosswhite have also indicated that the change has both internal and external components. The internal component consists of changes within the teacher and changes within the classroom directly under the teacher's control. The external component consists of changes within the structures that define the walls of the classroom (changes made by administration, the public, etc.--anything besides the teacher which affects the classroom). In both citations there is a clear message that the teacher can not make this change alone. The author, as a "reflective practitioner," believes that the teacher can be an important agent for change, however, in both the internal <u>and</u> the external areas.

The following discussion can be roughly divided into two parts. In the first two sections consideration is given to some of the personal areas of difficulty encountered by the author in implementing the lessons and some ideas about how a teacher can deal with them and find personal support when initiating a change of this sort. They deal mostly with the internal component of the changes which are necessary to create a facilitative classroom. The third section will deal with some of the external factors that impinge on the creation of that classroom.

Areas of Difficulty

Shared Control. Perhaps the greatest difficulty for this teacher continues to be that of shared control. The change from "the authority" to facilitator is not an easy one. It requires learning new skills, mainly skills that require the teacher to be more vulnerable and open to the student. This can create a great deal of discomfort for the teacher, particularly if the teacher has previously held the belief that the students will become unmanageable if the teacher is perceived as "vulnerable." The benefit of input from students is of great significance in helping a teacher deal with this issue. Reading the reflections of her students gave this author insight into how students feel about their classroom experiences. Their comments indicate that students want order, want to grow and learn, and appreciate being asked to take an active part in the process.

Piaget's idea that the purpose of education is to foster autonomy is helpful in dealing with this issue (Kamii, 1985, 1984). Students' behavior is difficult to manage if THEY view the responsibility for that control as emanating outside themselves. Traditional classrooms, in Piaget's view, foster this heteronomy, this view by the students that control exists by the teacher's authority. Piaget drew a distinction between affecting someone else's behavior by punishment (and reward) and using sanctions by reciprocity (Kamii, 1985, p. 43). Punishment for an undesired behavior is usually arbitrary and is not necessarily a direct consequence of the act.

An example of punishment would be to assign an hour of after school detention to a child whose behavior was disrupting his group's or class's attention to a task. A sanction of reciprocity, on the other hand, has a direct relation to the act and the adult's point of view. The disruptive child mentioned above would be given the choice of staying with the group and modifying his/her behavior or asked to leave the group to do some suggested activity alone UNTIL such time as the child decides he/she can behave in the group appropriately. Kamii describes this sanction as "exclusion from the group", and lists three others that can be used effectively; "appeal to the direct and material consequence of the act" ("When you act in that way, we cannot finish our task and that makes me feel frustrated."), "depriving the child of the thing he has misused", and "restitution" (1985, pp. 43,44). The teacher administers these sanctions in a non-punitive way and offers to help the child, for example, in making the restitution.

Student-Teacher Interaction. Chapter II contains some basic ideas that outline how the teacher needs to interact with the students to create the facilitative classroom. Allowing wait time after asking a question can significantly affect student responses. Asking a question, waiting for many students to volunteer a response before choosing one rather than calling on the first person to raise a hand can make a big difference in getting more of the students involved and obtaining thoughtful answers. In addition, if a student when called on does not know the "right" answer or any answer at all, knowing when to stick with that student, trying to help the student reveal his/her thinking in a way that does not embarrass the student, requires patience and skill on the part of the teacher. Helping a student explain his/her thinking, accepting whatever is said and trying to understand what thoughts led to a student's conclusion rather than moving on to someone else who is madly waving a hand or even vocalizing the "right" answer is a skill which requires practice on the teacher's part.

Another issue for the author in managing classroom discussions involved uncertainty about the amount of input the teacher should provide. An example of this uncertainty was caused while facilitating the discussion elicited in the lesson The Area Stays the Same, when students were asked to share their observations. If no student observes any of the important relationships between perimeter and compactness or elongation of shape, what should the teacher do or say? Duckworth has offered some useful advice in this regard.

> I do, though, offer ideas for consideration if I can see a different point of view that no one else has mentioned. Sometimes such an idea is one that I believe in, and sometimes it is not. In either case I do not present it as a "right" idea but simply as another one that should be considered. (I usually introduce it by saying, "Some people say..."). (Duckworth, 1986, p. 489)

What seems to the author as important in deciding these issues is to recognize that the teacher is one of the participants in the classroom dialogue and, as such, may offer ideas for consideration. This is not done as a special authority but rather in the way that any other participant might.

Structuring the Classroom. The teacher faces daily decisions about what activities to plan to enhance students' learning. Should all lessons arise from problem situations? Should students' work in partnerships and cooperative groups only? Should the teacher lecture or have kids work individually? These questions must be answered by each teacher in each classroom in a way that is comfortable for him/her.

It is only by experimenting with the use of different strategies that the teacher will discover what works most effectively for him/her or for his/her students. Again there is no single "right" way. What works today might not work with the next group. Teaching needs to be viewed as a fluid, creative process. The teacher is also a student acquiring new skills constantly. There is no "set of rules" to memorize that will enable the teacher to handle all situations perfectly, to anticipate all outcomes.

The author made the decision to begin the move to a more facilitative classroom by doing a problem-solving activity one day a week. The lessons selected from Marilyn Burns' tapes were implemented within this structure. Having a week to plan the next lesson, gather materials, and reflect on problems which arose was beneficial. This sort of lesson requires assembling materials, storing partly finished projects, passing out materials, and very different teacher interaction than what was customary. Since students, under the current school structure, are used to compartmentalization and fragmentation, the students were able to deal with the lessons when presented in this way. Time was lost in trying to pick up from where they had left off the previous week, however. Upon reflection the author concludes that students of this age might find it easier to deal with the demands of this type of unit if it were presented on consecutive days. In general, changes in structure in the middle school and in the mathematics classroom should be toward more integration and away from fragmentation. Developing a repertoire of problem-solving experiences such as these to present to students as the year progresses is now the author's goal.

Additional sources for lessons which satisfy the conditions set forth in the facilitative model outlined in Chapter II are beginning to appear. Marilyn Burns and Cathy McLaughlin have published a book of lessons for middle school (Burns, 1990). Michigan State University (1986) has published five units of lessons which present middle school mathematics concepts in an interrelated way. Development of concepts arises out of problem-solving contexts. Again, the components of the facilitative model set forth in the second chapter are well addressed.

At this point in time, the author does not believe that all lessons in a facilitative classroom will necessarily be of the sort described in Chapter IV. Cooperative learning is a wonderful tool but not efficient for presentation of all material. A combination of lecturing, partner and group interaction, individual work, group discussion, and whole class discussion are all available in designing lesson plans. The facilitative model does lead to new guidelines for managing even the most traditional way of presenting new material--the lecture.

In designing lectures it is important to recognize that they can be made more interactive. If the teacher has presented a new concept in a mathematics classroom, it is often followed by giving students a chance to practice the concept or skill. The results of this practice session can be handled using the following guidelines. Rather than asking for a student's answer and responding right or wrong, a teacher can just write the student's answer down with no comment and ask if there are any other answers. If there are many different answers, the teacher may ask each of the students for the process, or thinking the student used to arrive at the answer. This will often enable a student to uncover his/her error. The student who has the correct answer and can explain the method often enables the student with the incorrect answer to identify where he/she went wrong. Often, when the student with the incorrect response explains his/her thinking the cause of the incorrect response will be discovered by the student him/herself or someone else in the class. The important key is to give each student ample opportunity to uncover errors on his/her own. Again, clear recognition should be given to the fact that making errors is inevitably part of the learning process and that each individual has the capability to uncover and correct his/her own erroneous thinking.

Uncertainty of Lesson Outcome. It is important to recognize that a lesson plan is only a prediction of what the teacher thinks will happen. A part of this idea of being open and flexible is to expect that the teacher can never anticipate where a particular lesson might lead. Each group of students will deal with the lesson in a different way. The teacher needs to recognize that this is part of the process; that it does not mean that the lesson did not work. In attempting to guide this sort of lesson, it is extremely important that the teacher allow him/herself the same consideration as is to be afforded to the other students in the room. The basic components of this consideration entails a recognition that the process of learning naturally involves: 1) making mistakes, 2) belief that all students are capable of discovering and correcting errors, and 3) that making mistakes is not something for which one is criticized. An important connection is that the very process that the teacher is trying to institute for the student should of necessity be applied to the self.

Creating Mechanisms for Support

This section deals with considerations the teacher needs to make in providing means for him/herself that support the change to a reflective learner managing a facilitative classroom.

Mechanism for Reflective Review. The teacher should create for him/herself some sort of mechanism for reflection on the process. There is not one right way to do this. Some suggestions: videotaping, or tape recording some lessons for later analysis. In reviewing these what the teacher needs to keep in mind are the questions: What went well? What did I like about how I handled that lesson? What did not go well? How might I handle a similar situation differently in the future?

Some teachers might find it more helpful to keep a daily journal of reflections on the lessons. This could include a consideration of the above questions as well.

Peer Discussion. The importance of communication with others as a part of the learning process is central to the constructivist view of learning. The current structure of most public schools does not afford teachers the opportunity for collegial support. In fact Flinders argues that teacher isolation is a natural outgrowth of the current school structure and, in addition, "is an adaptive strategy for teachers because it protects the time and energy required to meet immediate instructional demands" (Flinders, 1988, p. 25). Flinders makes the point that any attempt at school reform must take this into consideration.

Since change of the sort discussed in this thesis is difficult, if not impossible, without collegial support and interaction, ultimately the best way for our classrooms to change is to change the structure of the teacher's working situation so that time and opportunity for peer support is made available (an external change, of course). Short of this sort of change in the working day, what can an individual teacher do? Some teachers may find it useful to work with other like-minded teachers in instituting these changes. They can set aside time to share difficulties encountered or make an arrangement for mutual observation for the purpose of constructive feedback. If support is not directly available in a teacher's school, having opportunities to discuss experiences with other teachers with the same philosophical beliefs can also be supportive and helpful. Teacher conferences, local universities, and teacher association meetings are all possible places to find contacts among people who are trying to move the classroom in the direction of our new model. The key here is to recognize that some sort of support is beneficial and to be creative in finding ways to seek it out.

This author found the videotape recordings of the lessons used for this thesis could be approached in the same way as any other student's attempt at developing a new skill. They afforded an opportunity to become an observer and reflect on the progress of the changes in the teacher interactions in the classroom. Much of the author's support came from the teachers and students in the Critical and Creative Thinking program (in which she was enrolled at UMass-Boston). This was reinforced by the teachers encountered at mathematics conferences and in other courses such as Teacher Effectiveness Training.

The difficulty of being a "reflective practitioner" (Schon, 1983) is recognized by Eleanor Duckworth:

> It is a rare schoolteacher who has either the freedom or the time to think of her teaching as research, since much of her autonomy has been withdrawn in favor of the policies set by anonymous standard setters and test givers.
>
> But even given the terrible constraints, and even if no resources are available to make known what they learn, there is some opportunity--and I think great need--for teachers to listen to their students explain what they think. (Duckworth, 1986, p. 494)

As the author has stated before, the transition to a facilitative classroom will not come quickly or easily, but the author believes that the attempt to make this transition, although it leads the traditional teacher to feelings of frustration, failure, and uncertainty at times, is also a path to personal growth. The process of trying to make this move is the process of learning. The teacher as a learner leading other learners is the model this author will continue to carry into her seventh grade classroom. The teacher-learner must search for opportunities to enhance learning both inside and outside of that classroom. The next section will consider external factors that affect the creation of the facilitative classroom.

External Support for the Facilitative Classroom

Inevitably, any teacher struggling to incorporate the changes discussed in this thesis must recognize that some of the constraints on that classroom must also change if the facilitative classroom is to become a reality. The following will reconsider the constraints delineated in the first and third chapters and reflect on the effect each has on creating a more facilitative classroom.

Changing Classroom Constraints. First to be considered are the constraints of time and structure. Rigid, 45 minute periods and compartmentalized study are not the best arrangement for students to recognize the personal relevance of mathematics in their lives. Other arrangements of the school day and of subject presentation would be extremely desirable. The walls of the traditional classroom must be made more flexible to enable the transition to a more facilitative one.

Teachers of mathematics and science, mathematics and social studies, or mathematics and art--working together--could develop interesting units through which students could experience the reality of the interaction of these subjects in solving problems of everyday life and in grappling with the issues we face as a society. This calls for a significant change in the way most middle schools are structured. Common time set aside for planning, teachers responsible for a common pool of youngsters, and a more flexible scheduling of period length would greatly enhance the possibilities of planning lessons which help students develop thinking and mathematical skills in context. The rigid structure of the day and the 45 minute period means that any lessons involving extensive use of materials are difficult to manage. Time must be spent at the start and end of each period getting materials in and out of storage. Scheduling double periods would greatly improve this situation. The teacher and students often feel pressed, rushed, interrupted--the current schedule is not conducive to thoughtful, perseverant work on the part of the student.

Curricula. The author mentioned in the previous section "Structuring the Classroom" that sources of interesting lessons are certainly beginning to appear on the scene. Marilyn Burns published last year a book containing lessons for middle school. A teacher desiring to make the classroom more facilitative will probably draw lessons from sources such as these rather than ever relying exclusively on a textbook for the source of all lessons. No discussion of new curricula can ignore the massive undertaking at the University of Chicago under the direction of Zalman Usiskin to create a new mathematics for kindergarten through twelfth grade.

Assessment. Alternative ways of assessing students' mathematical development are emerging for use in both the classroom and as extensions of standardized tests. A thorough analysis of these methods is beyond the scope of this thesis, but the work being carried out in many places to find new ways to access students is encouraging. The NCTM has established standards for student assessment which differ from the traditional paper and pencil tests of most classrooms and differ from the multiple choice approach of the standardized test. Such ideas as portfolios of student work which are carried along throughout a student's school career, holistic scoring, structured interviews, open ended problem solving are but a few of the ideas being put forth by such groups as the NCTM (1987), the California Mathematics Council (1989), and the Massachusetts Department of Education (1990).

Continuum of Experiences. Although it is possible for a teacher to initiate the change to a more facilitative model in a single classroom, that teacher will be continually frustrated unless that classroom is part of a continuum of classrooms moving in a more facilitative direction. A system-wide dedication to a more facilitative approach to mathematics education can be addressed by teachers. The movement to restructure schools so the teachers in the classroom have more input into the structure of the whole system goes hand in hand with the changes herein outlined for the classroom. Teachers need to become active in pushing for changes in structure within the system. Reference to the thrust of the NCTM Standards can be used to support the need for these changes when dealing with administration and the local public.

Beliefs and Attitudes. In the first chapter the author listed several factors contributing to the current state of mathematics education. Factors of student-teacher interaction, curriculum, and classroom structure have been addressed in these reflections. The factor most difficult to change is current beliefs and attitudes about mathematics and its personal relevance and accessibility. This author sees the struggle in which she is engaged in light of a larger set of goals, to restructure mathematics education to make this powerful aspect of human endeavor, "...one of the most powerful and adaptable mental tools which the intelligence of man has made for its own use" (Skemp, 1987, p. 6), accessible to the majority of our students. These beliefs and attitudes can be affected by an accumulation of positive experiences in mathematics classrooms. The movement has begun, the movement toward a more facilitative approach. The challenge is there for all of us engaged in mathematics education, to encourage and support each other as we undertake the difficult task of removing the walls of the traditional classroom and enabling our students' access to the power of mathematics.

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APPENDIX

SQUARE UNITS

On a piece of graph paper draw the following rectangles to show their area. Use the distance between two lines as one unit. Write the area of each rectangle inside it. Write the perimeter under each rectangle.

1. Length: 10 inches
   Width: 5 inches

2. Length: 7 feet
   Width: 6 feet

3. Draw on the graph paper all of the different rectangles you could make with 36 tiles. Record the perimeter of each. What is the area of each?

4. Draw a square yard on the graph paper. Have the distance between the lines on the graph paper represent one foot. How many square feet in your square yard? (No! It is NOT 3 square feet--count them.)

5. Draw a square foot on your graph paper. Have the distance between the lines on the graph paper represent one inch. How many square inches in your square foot? (It's a lot more than 12!)

6. Use the results from questions 4 and 5 to figure out the number of square inches in one square yard.

REVIEW
AREA AND PERIMETER

Use graph paper to draw shapes for each problem. Assume the lines on the graph paper are one unit apart. Answers must have the correct units.

1. Draw and calculate the area and perimeter of an 8 foot square. Record the area inside the square, the perimeter below the square.

2. What is the area of the square in question 1 in square inches? What is the perimeter in inches?

3. If I have 24 square tiles one foot by one foot in size, what are the dimensions of the rectangle with the greatest perimeter I can make? Least perimeter?

4. If John has 40 feet of fencing, what is the greatest rectangular area he can fence in for his garden?